Convective Transport over an Exponentially Stretching Surface

A dissertation submitted to the
NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA (ODISHA)
in partial fulfillment of the requirements
for the award of the degree of
MASTER OF SCIENCE
in
MATHEMATICS
by

SUMITRA SUTAR

under the guidance of
Dr. CH. RAMREDDY

Department of Mathematics
National Institute of Technology
Rourkela-769008
India

May 2013

D E C L A R A T I O N

I hereby certify that the work which is being presented in the dissertation entitled " **Convective Transport over an Exponentially Stretching Surface** " in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology Rourkela, is the bonafide work carried out under the supervision of Dr. CH. RAMREDDY. The contents of this dissertation have not been submitted elsewhere for the award of any degree.

.
.

(SUMITRA SUTAR)
(Roll No-411MA2085)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Dr. CH. RAMREDDY
Assistant Professor
Department of Mathematics
National Institute of Technology,
Rourkela, Odisha, INDIA

.

ACKNOWLEDGEMENTS

I deem it a privilege and honour to have worked in association with Dr. Ch. Ramreddy, Assistant Professor, Department of Mathematics, National Institute of Technology Rourkela, India. I express my deep sense of gratitude and indebtedness to him for suggesting me the problem and guiding me throughout the dissertation. Words are inadequate to express my feelings of thankfulness for all the parental care and affection he has shown while my work was in progress.

I express my special thanks to Prof. G.K. Panda, Head, Department of Mathematics, N.I.T. Rourkela, for his valuable suggestions and affection lavished on me and providing necessary help and support throughout the period of my study here.

I take this opportunity to thank Prof. A. Behera, Prof. D.G. Sahoo, Prof. K.C. Pati, Prof. S. Chakraverty, Dr. Raja Sekhar, Dr. Jugal Mohapatra and all other faculty members of the department for their encouraging words from time to time during my course work. Also, I am grateful to office staffs, Department of Mathematics for their kind support.

I thank the Director, National Institute of Technology Rourkela, for providing the facilities to pursue my postgraduate degree.

I thank all my classmates and friends for making my stay memorable at National Institute of Technology Rourkela.

Finally, I also thank my parents for their constant inspiration.

. SUMITRA SUTAR

A B S T R A C T

The prediction of heat and mass transfer characteristics about mixed convection over an exponentially stretching surface in Newtonian fluids is very important due to its practical engineering and industrial applications, such as glass-fiber production, wire drawing, paper production, plastic sheets, metal and polymer processing industries and many others. Realizing the increasing technical applications of magnetohydrodynamics (MHD) effects, the problem of MHD boundary layer flow over an exponentially stretching sheet has been studied by a number of researchers. The Newtonian fluid flow equations constitute a coupled system of equations and are complicated. In spite of this, in this dissertation an attempt has been made to study the steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over an exponentially stretching sheet in the presence of cross-diffusion effects.

The dissertation consists of THREE chapters. The first chapter (i.e. Chapter - 1) provides an introduction to the concepts in Newtonian fluid, MHD, heat and mass transfer and a review of the relevant literature. The second chapter (i.e. Chapter - 2) deals with mixed convection heat and mass transfer over an exponentially stretching surface embedded in the Newtonian fluid in the presence of MHD, Soret and Dufour effects.

In all the cases of the chapter the governing equations are transformed into a set of nonlinear non-homogenous differential equations using the local-similarity transformations. The transformed boundary layer ordinary differential equations are solved numerically using the shooting and matching technique. The effects of MHD parameter, Soret and Dufour numbers together with mixed convection parameter on the velocity, temperature, concentration, skin friction coefficient, heat transfer rate and mass transfer rate are considered.

The last chapter (i.e. Chapter - 3) gives a summary and overall conclusion.

NOMENCLATURE

$\mathcal{B}$	Buoyancy ratio.
B_0	Strength of the magnetic field.
C	Concentration.
C_f	Skin friction coefficient.
C_p	Specific heat capacity (isobaric).
C_s	Concentration susceptibility.
C_w	Wall concentration.
C_∞	Ambient concentration.
D	Solutal diffusivity.
D_f	Dufour number.
f	Reduced stream function.
g^*	Gravitational acceleration.
Gr, Gc	Thermal, Solutal Grashof numbers respectively.
Ha^2	Hartman number.
M	Magnetic parameter.
k	Thermal conductivity.
K_T	Thermal diffusion ratio.
L	Characteristic length of the plate.
Nu_x	Local Nusselt number.
Pr	Prandtl number.
q_w, q_m	Heat, Mass fluxes
Re	Reynolds number
Re_x	Local Reynolds number
Ri	Mixed convection parameter
Sc	Schmidth number.
Sh_x	Local Sherwood number.
S_r	Soret number.
T	Temperature.
T_m	Mean temperature.
T_w	Wall temperature.
T_∞	Ambient temperature.
U_*	Characteristic velocity.
u_∞	Free stream velocity.
u, v	Velocity components in x and y directions.
X	$X-$ location.
x, y	Coordinates along and normal to the plate.
α	Thermal diffusivity.
β_T, β_C	Coefficients of thermal and solutal expansion.
η	Similarity variable.
θ	Dimensionless temperature.
ϕ	Dimensionless concentration.
μ	Dynamic viscosity.
ν	Kinematic viscosity.
ρ	Density of the fluid.
σ	Electrical conductivity of the fluid.
τ_w	Wall shear stress.
ψ	Stream function.

Subscripts

w	Wall condition.
∞	Ambient condition.
C	Concentration
T	Temperature

Superscript

$'$	Differentiation with respect to η.

Contents

Chapter 1

Motivation

1.1 Introduction

Fluid Dynamics is a branch of science, which is concerned with the study of the motion of fluids or that of bodies in contact with fluids. It has applications in Astrophysics, Biophysics, Geophysics, Meteorology, Oceanography, and Atmospheric sciences. Besides its applications in basic sciences, it has also applications in Civil engineering for designing dams, irrigation canals, and water supply systems, Mechanical and Nuclear engineering for designing turbines, pumps, compressors, heat exchangers, cooling system devices, electro-chemical devices and fluid couplings, Chemical and Petroleum engineering for designing efficient devices to mix industrial chemicals and in filtering the chemicals, oils, and petrol and Aerospace engineering for designing airplanes that have low resistance and at the same time have high lift force to support the weight of the aeroplane.

Like the study of any other branch of science, Fluid dynamics needs mathematical analysis as well as experimentation. Fluid dynamics via experimentation is studied under disciplines like hydraulics, and Aeronautical engineering etc, while the mathematical approach to study the behavior of fluid is undertaken under the field Computational Fluid Dynamics (CFD). CFD in the literature is defined, as it is the prediction important branch of the effects of fluid motion past or through objects by numerical methods rather than model experiments. CFD is a very important branch now a days because of the non-linear nature of the governing equations of fluids and the complexity of the solutions required.

A Newtonian fluid is a fluid that flows like water-its stress versus rate of strain curve is linear and passes through the origin. A non-Newtonian fluid is a fluid in which the viscosity changes with the applied strain rate. As a result, non-Newtonian fluids may not have a well-defined viscosity. Further, Newtonian fluids are those that obey Newton's law relating shear stress and shear rate with a simple material property (the viscosity) dependent on basic thermodynamic variables such as temperature, concentration and pressure, but independent of flow parameters such as shear rate and time. Convective heat and mass transfer in a "Newtonian fluid" have been extensively investigated during the last several decades including different physical effects.

In spite of a complex nature and complex interactions of Newtonian fluid with the flow, boundary-layer assumptions were successfully applied to this model and over the last few decades much work has been done on them for a large variety of body geometries. Further, a number of studies have been reported in the literature on convective heat and mass transfer in a Newtonian fluid. Realizing the increasing technical applications of magnetohydrodynamics (MHD) effects, in this dissertation an attempt has been made to study the combined effects of MHD and cross-diffusion on the mixed convection problems in a Newtonian fluid.

1.2 Heat and Mass Transfer

The heat transfer is a science that determines how and at what rate heat energy is transferred as a result of a temperature gradient or differences. Heat transfer occurs in three modes *Conduction*, *Convection* and *Radiation*. In *Conduction*, heat transfers from the region of higher temperature to a region of lower temperature by kinetic motion or direct impact of the molecules whether the medium is at rest or in motion. *Convection* is a mechanism in which the heat transfers because of the movement of fluid from one region to the other region in the medium. The conversion of the internal energy of a substance into radiant energy is referred to as *radiation* heat transfer. Pure conduction can be observed in solids where as heat transfer by convection is always accompanied by conduction and this is observed in fluid media. This combined process of heat transfer by conduction and convection is referred to as convective heat transfer. Convective heat transfer is further classified as *Forced Convection*, *Free Convection* and *Mixed Convection*. *Forced Convection* is due to an external agent unrelated to heating effects, which induces the flow of fluid over the heated body. *Free or Natural Convection* flow is caused by natural forces such as buoyancy forces which arise from density differences in a fluid. These density differences are the consequence of temperature and concentration gradients within the fluid. In *Mixed Convection* the order magnitude of the buoyancy force is comparable to the externally maintained pressure drop to force the flow.

In order to estimate the heat transfer rate in the medium, we need to determine the temperature distribution or the temperature field. The temperature field is determined by solving the heat equation, which is a statement of conservation of energy or the first law of thermodynamics. Using the local equilibrium model, the principle of conservation of energy gives

$$\sigma \frac{\partial T}{\partial t} + V.\nabla T = \nabla.(\alpha \nabla T) \tag{1.1}$$

where T is the local equilibrium temperature, σ is the heat capacity ratio and α is the thermal diffusivity of the medium.

The term mass transfer means the tendency of a component in a mixture to travel from a region of high concentration to one of low concentration. There are two basic modes of mass transfer: *Diffusion* and *Convection*. There is a close similarity between heat transfer and mass transfer in terms of the transport rate equation and transport conservation equation. The convection mass transfer is analogous to convection heat transfer and occurs between a moving mixture fluid species and an exposed solid surface. Like heat transfer rates, the species mass flux can be determined from the mass conservation field by solving the species mass conservation equation, which is a statement of conservation of mass species. The conservation of mass is given by

$$\epsilon \frac{\partial C}{\partial t} + V.\nabla C = \nabla.(D \nabla C) \tag{1.2}$$

where D is the solutal diffusivity and ϵ is the porosity. We note that this equation is similar to eqn. (1.1).

1.3 Soret and Dufour Effects

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients.

Thermal diffusion, also called thermodiffusion or Soret effect, corresponds to species differentiation developing in an initial homogeneous mixture subjected to a temperature gradient ([24]). The heat flux induced by a concentration gradient is called Dufour or diffusion-thermo effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. For instance, Eckert and Drake [7] presented several cases of Dufour effect where as the Soret effect has been utilized for isotope separation and in a mixture between gases with very light molecular weight and of medium molecular weight.

A very useful reference on thermodiffusion in porous media is that by Saghir *et al.* [21], wherein there is an extensive literature review on the measurement techniques of the Soret coefficient and on the numerical work done in this area.

In a general form the energy and concentration equations are expressed in the form

$$\frac{(\rho c)_m}{(\rho c)_f}\frac{\partial T}{\partial t} + V.\nabla T = \nabla.\left(\alpha \nabla T + D_{TC}\nabla C\right) \tag{1.3}$$

$$\epsilon \frac{\partial C}{\partial t} + V.\nabla C = \nabla.\left(D\nabla C + D_{CT}\nabla T\right) \tag{1.4}$$

(see Nield and Bejan [14]), where D_{TC}/α and D_{CT}/D may be considered as Dufour and Soret coefficients (numbers) of the medium.

1.4 Magnetohydrodynamics

When a conductor carrying an electric current moves in a magnetic field, it experiences a force tending to move it at right angles to the electric field. Conversely, when a conductor moves in a magnetic field, a current is induced in the conductor in a direction mutually at right angles to both the field and the direction of motion. These two statements, first enunciated by Faraday, constitute the laws of electromagnetism. In the case when the conductor is either a liquid or a gas, electromagnetic forces is generated which may be of the same order of magnitude as the hydrodynamics and inertial forces. Thus the equations of motion take these electromagnetic forces (Lorentz forces) into account as well as the other forces. The science which treats these phenomena is called magneto-hydrodynamics (MHD) or magneto-fluid dynamics or magneto-gas dynamics, etc.

There are two key physical effects which occur when the fluid moves into the magnetic field: motion of a conducting liquid in an applied magnetic field will induce a magnetic field in the medium. The total field is the sum of the applied and induced magnetic fields ($\bar{B} = \bar{B_0} + \bar{b}$, $\bar{b}$ is induced magnetic field). The relative strength of the induced field is characterized by the magnetic Reynolds number. The neglect of the induced magnetic field is a valid assumption when the magnetic Reynold's number is small. The second key effect is dynamical. When currents are induced by a motion of a conducting fluid through a magnetic field, a force (Lorenz force or $\bar{J} \times \bar{B}$, where $\bar{J}$ is the current density and $\bar{B}$ is the magnetic field) will act on the fluid and alter its motion. Hence, fluid motion in a magnetic field induces electric currents, which alter the flow field, and at the same time, the flow of electric currents in the magnetic field produces a mechanical force that brings a change in the fluid motion. Suppose that the fluid is incompressible, electrically conducting and is in the presence of an arbitrary magnetic field. The magnetic field then interacts with the fluid by means of body force and body couple per unit mass. If gravitational effects are not present, then a regular magneto-fluid dynamics assumption is $\rho\bar{f} = \rho_e E + \bar{J} \times \bar{B}$, where ρ_e is the free charge density. Since, the electric force density $\rho_e E$ is smaller than $\bar{J} \times \bar{B}$, the electric force density can be neglected. Hence, the fluid dynamical aspects of MHD are handled by adding an electromagnetic force term to the momentum equation of the fluid.

1.5 Boundary Conditions

Various types of boundary conditions used in the study of convective heat and mass transfer in a Newtonian fluid (for boundary layer and enclosure flows) are discussed in Nield and Bejan [14]. In this study, we used the exponentially stretching velocity at the boundary and impermeability of the vertical surface. Also, the vertical surface is (i) maintained at an exponentially stretching temperature and concentration and (ii) subject to an exponentially stretching heat and mass flux conditions. Outside the boundary layer, the flow in the mainstream direction is assumed to be zero in the mixed convection flows. In practice, the temperature and concentration of the ambient fluid far away from the surface, T_∞ and C_∞, may be taken as constants(isothermal) respectively. Further, the wall temperature and concentration are assumed to be greater than the ambient temperature and concentration.

1.6 Shooting Method

The governing equations of convective heat and mass transfer in Newtonian fluids are essentially nonlinear ordinary or partial differential equations. Generally, these nonlinear differential equations cannot be solved analytically, so recourse must be made to a numerical approach. However no single numerical method is applicable to every nonlinear differential equation. Some of the popular methods that are available to solve these nonlinear differential equations are quasi-linearization, shooting methods, local similarity and non-similarity methods, finite element methods, cubic spline collocation method, Keller-box method etc.

Among these, the shooting method is an efficient and accurate numerical scheme for the ordinary differential equations. This method has several desirable features that make it appropriate for the solution of all parabolic differential equations. Hence, the system of reduced nonlinear ordinary differential equations together with the boundary conditions are similar and solved numerically using fourth-order Runge-Kutta scheme with a shooting technique. The method has the following five main steps:

i. This method replaces the given BVP by a sequence of initial value problems (IVPs) for the same ODE with initial conditions;

ii. Integrating the sequence of IVPs using fourth-order Runge-Kutta scheme;

iii. Identifying the initial slopes for the missing conditions using Newton-Raphson method;

iv. The integration length varies with the parameter values and it has been suitably chosen each time such that the boundary conditions at the outer edge of the boundary layer are satisfied;

v. Repeating this procedure till the convergence is obtained satisfying the boundary conditions.

This method has been proven to be adequate for boundary layer equations, seen to give accurate results and has been widely used. It seems to be the most flexible of the common methods. The scheme is also applicable to various types of boundary layer flow problems, including the free and mixed convection flows.

1.7 Literature Review

Mixed convection flows or combined forced and free convection flows arise in many transport processes in engineering devices and in nature. These flows are characterized by the buoyancy parameter $Ri = Gr/Re^m$ where Re is the Reynolds number, Gr is the Grashof number and $m(> 0)$ is a constant which depends on the flow configuration and the surface heating conditions. The mixed convection regime is generally defined as the range of $a \leq Gr/Re^m \leq b$ where a and b are the lower and the upper bounds of the regime of mixed convection flow respectively. The parameter Ri provides a measure of the influence of the free convection in comparison with that of forced convection on the fluid flow. Outside the mixed convection region, $a \leq Gr/Re^m \leq b$, either the pure forced convection or the pure free convection analysis can be used to describe accurately the flow or the temperature and concentration fields. Forced convection is the dominant mode of transport of heat when $Gr/Re^m \to 0$, whereas free convection is the dominant mode when $Gr/Re^m \to \infty$, or alternatively $Gr^m/Re \to \infty$(See Chen and Armaly [5]). Buoyancy forces can enhance the surface heat transfer rate when they assist the forced flow and vice versa. Buoyancy forces also play a significant role in the incipience of flow instabilities and they can be responsible for either delaying or speeding up the transition from laminar to turbulent flow.

The analysis of mixed convection boundary layer flow along a different surface geometries embedded in viscous fluid has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Extensive studies of mixed convection heat and mass transfer of a non-isothermal vertical surface under boundary layer approximation for Newtonian fluids have been undertaken by several authors. Further, the problem of mixed convection boundary layer on vertical plate in viscous fluids for both isothermal and uniform heat flux cases has received much attention in the past. A very long list would be required to exhaust the published literature for the problem of steady boundary-layer mixed convection over a vertical surface. An exhaustive review of the literature on mixed convective heat and mass transfer can be found in the book in Bejan [3].

The study of flow, heat and mass transfer in the boundary layer of a continuously stretching surface with a given temperature distribution moving in an otherwise quiescent fluid medium has attracted the attention of researchers for the past few decades due to its numerous industrial and engineering applications. After the pioneering works of Sakiadis [23], several researchers discussed the problem of boundary layer flow to obtain the thermal and kinematic behavior by considering the different forms of stretching velocity. Magyari and Keller [13] discussed the similarity solutions of flow and thermal boundary layers on an exponentially stretching surface. These solutions involve an exponential dependence of the temperature distribution in the direction parallel to that of the stretching. Partha *et al.* [16] presented a similarity solution for mixed convection flow and heat transfer from an exponentially stretching surface by considering the viscous dissipation effect in the medium. Elbashbeshy [8] examined the flow and heat transfer characteristics over an exponentially stretching continuous surface with suction. The effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet has been solved analytically by Sajid and Hayat [22] and numerically by Bidin and Nazar [4].

In the recent past, considerable attention has been paid to the theoretical and numerical study of these combined effects as it is considered as second order phenomena and are significant in areas such as hydrology, petrology and geosciences. Due to the importance of Soret and Dufour effects, several authors worked on the problems with different surface geometries. Although, the Soret and Dufour effects of the medium on the convective transport in a viscous fluid are important, very little work has been reported in the literature. The Dufour effect was recently found to be of order of considerable magnitude such that it cannot be neglected [Eckert and Drake [7]]. Dursunkaya and Worek [6] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [11] presented the same effects on mixed convective and mass transfer transfer, steady, laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Several researchers have reported combined effects of Dufour and Soret in the mixed convection flows. Both free and forced convection boundary layer flows with Soret and Dufour have been addressed by Abreu *et al.* [1]. The effect of Soret and Dufour parameters on free convection heat and mass transfers from a vertical surface in a doubly stratified Darcian porous medium has been reported by Lakshmi Narayana and Murthy [12]. A mathematical model for the steady thermal convection heat and mass transfer in a micropolar fluid saturated Darcian porous medium in the presence of significant Dufour and Soret effects and viscous heating is presented by Rawat and Bhargava [20]. Recently, Srinivasacharya and RamReddy ([27]; [29]) explained the importance of Soret and Dufour effects on mixed convection in a micropolar fluid.

There has been a renewed interest in MHD flow, heat and mass transfer in viscous and clear domains due to the important effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluid such as MHD power generators, the cooling of nuclear reactors, plasma studies, purification of molten metals from non-metallic inclusion, geothermal energy extractions etc. Accordingly, it is of interest to examine the effect of the magnetic field on the flow, heat and mass transfer characteristics. Studying such an effect has great importance in various application fields where MHD, Soret and Dufour effects are correlative. Hence, the coupled heat and mass transfer problems in the presence of a magnetic field and Soret and Dufour effects received a considerable amount of attention. Many problems of MHD, Soret and Dufour effects in viscous fluids by considering different surface geometries have been analyzed and reported in the literature. For example, Raptis and Singh [19] have solved the problem of hydromagnetic natural convection flow past an accelerated vertical plate. Postelnicu [17] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam *et al.* [2] analyzed the Dufour and Soret effects on steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. Recently, the effect of MHD, double dispersion and double stratification effects on natural convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated non-Darcy porous medium has been investigated by Srinivasacharya *et al.* [[25]; [26]].

1.8 Aim and Scope

The aim of the present dissertation is to study mixed convection boundary layer flows in electrically conducting Newtonian fluid in the presence of combined Soret and Dufour effects. The problems considered in the dissertation deal with exponential surface geometry.

1.9 Outline of the Dissertation

This dissertation consists of 3 chapters.

Chapter - 1 is introductory in nature and gives motivation to the investigations carried out in the dissertation. A survey of pertinent literature is presented to show the significance of the problems considered. The basic equations governing the flow, heat and mass transfers of Newtonian fluid are given.

Chapter - 2 deals with the problem of mixed convection heat and mass transfer near an exponentially stretching surface in a viscous fluid in the presence of magnetic field, and Soret and Dufour effects. In this chapter, the nonlinear governing equations and their associated boundary conditions are initially cast into a dimensionless form using local-similarity transformations. The resulting system of equations is then solved numerically by using a shooting method. The obtained numerical results are compared and found to be in good agreement with previously published results in special cases. The effects of magnetic and mixed convection parameters, Soret and Dufour numbers and also X-location are examined and are displayed through graphs. In addition, the skin-friction coefficient, Nusselt number and Sherwood number are presented in a tabular form.

In the above chapter, two types (cases) of boundary conditions for the temperature and concentration on the vertical surface are considered. In the first type of boundary condition, it is performed that the wall temperature and concentration are exponentially distributed on the surface. The second type of boundary condition involves the imposition of exponentially distributed heat and mass fluxes on the surface. In both cases, the usual conditions on velocity components are prescribed.

In Chapter - 3, the main conclusions of the earlier chapters are recorded and the directions in which further investigations may be made are indicated.

List of references is given at the end of the dissertation. The references are arranged in an alphabetical order. As a broad review of the existing literature is presented in the introductory chapter itself, in the chapter only a brief introduction to the concerned problem is given.

Considerable part of the work in the dissertation is communicated for publication in reputed journals.

Chapter 2

Mixed convection in the boundary layers on an exponentially stretching surface with MHD and cross-diffusion effects

2.1 Introduction

The analysis of mixed convection boundary layer flow along an exponentially stretching surface embedded in viscous fluid has received considerable theoretical and practical interest. Several authors have studied the problem of mixed convection about different surface geometries. For an exhaustive discussion of the mixed convection heat transfer in the boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution, the reader is referred to the works of Bidin and Nazar [4] (also see the references cited therein). Realizing the increasing technical applications of magnetohydrodynamics (MHD) effects, the problem of MHD boundary layer flow over an exponentially stretching sheet has been studied by a number of investigators. An analysis has been carried out by Pal [15] to describe the mixed convection heat transfer in the boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution in the presence of a magnetic field, viscous dissipation and internal heat generation/absorption. Ishak [9] studied the thermal radiation effects on hydro-magnetic flow due to an exponential stretching sheet. Recently, the steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over an exponentially stretching sheet in the presence of a uniform transverse magnetic field with viscous dissipation and radiative heat flux is discussed by Jat and Gopi Chand [10]. But, in some/or all the studies they have neglected buoyancy effects.

The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. These effects are considered as second order phenomena and are significant in areas such as hydrology, petrology and geosciences. Although the Soret and Dufour effects of the medium on the heat and mass transfer along the exponentially stretching surface in a fluid medium are important, very little work has been reported in literature. The Soret and Dufour effects on mixed convection flow, heat and mass transfer from an exponentially stretching vertical surface in a quiescent fluid is analyzed by Srinivasacharya and RamReddy ([18]; [28]; [30]; [31]).

Motivated by the above-mentioned works, and considering the importance of magnetic field, and Soret and Dufour effects for flow in a quiescent medium, the problem of mixed convection heat and mass transfer near an exponentially stretching surface in a viscous fluid is investigated. To our best knowledge and from the literature, till date, there is no study which has considered this problem. This study is an extension of the works by Srinivasacharya and RamReddy ([28]; [30]) to include magnetic effects. Using local-similarity variables, the nonlinear governing equations and their associated boundary conditions are converted into non-dimensional forms and hence the reduced system of equations is then solved numerically using the shooting and matching technique. The effects of magnetic, mixed convection parameters, Soret and Dufour numbers and also X-location are examined and are displayed through graphs. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2.2 Mathematical Formulation

Consider a steady, two-dimensional and laminar flow of an electrically conducting and incompressible viscous fluid near an impermeable plane vertical wall stretching surface. The free stream velocity which is parallel to the exponentially stretching surface is u_∞, temperature is T_∞ and concentration is C_∞. The physical model and coordinate system are shown in Fig.(2.1). The x-axis is directed along the continuous stretching surface and points in the direction of motion. The y-axis is perpendicular to plate and to the direction of the slit (z-axis) whence the continuous stretching plane surface issues. A uniform magnetic field B_0 is assumed to be applied in the y-direction. It is assumed that the induced magnetic field of the flow is negligible in comparison with the applied one which corresponds to a very small magnetic Reynolds number. We assume that the Soret and Dufour effects are present.

Using the Boussinesq and boundary layer approximations, the governing equations for the viscous fluid in the presence of Soret and Dufour effects ([11]) are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g^*\left(\beta_T(T-T_\infty)+\beta_C(C-C_\infty)\right)-\frac{\sigma B_0^2 u}{\rho}, \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{DK_T}{C_sC_p}\frac{\partial^2 C}{\partial y^2}, \tag{2.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{DK_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{2.4}$$

where u and v are velocity components in x and y directions respectively, T is the temperature, C is the concentration, g^* is the acceleration due to gravity, ρ is the density, B_0 is the strength of the magnetic field, σ is the electrical conductivity of the fluid, μ is the dynamic coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansions, α is the thermal diffusivity, D is the solutal diffusivity of the medium, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature and K_T is the thermal diffusion ratio. The last terms on the right-hand side of the energy equation (2.3) and diffusion equation (2.4) signify the Dufour or diffusion-thermo effect and the Soret or thermal-diffusion effect. The same symbols are used throughout the dissertation unless otherwise specified.

In this chapter, two types (cases) of boundary conditions for the temperature and concentration on the exponentially stretching surface are considered. In case (a), the surface is maintained at an exponentially stretching temperature and concentration. In case (b), the surface is subjected to an exponentially stretching heat and mass fluxes.

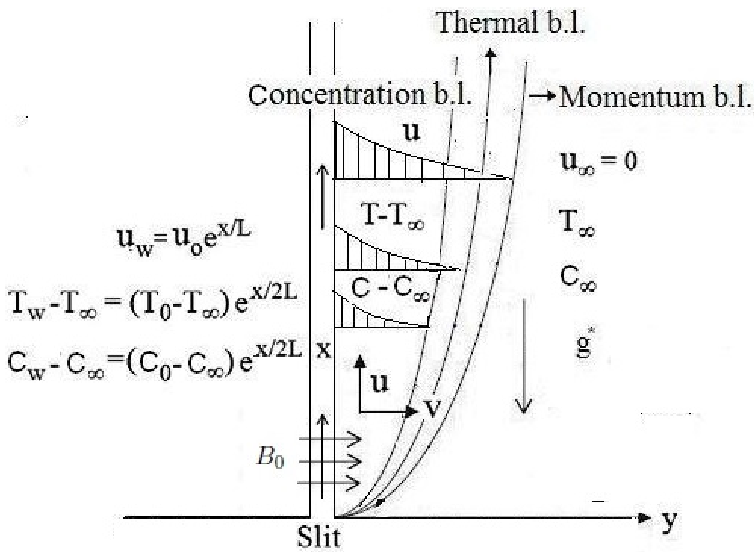


Figure 2.1: *Physical model and coordinate system.*

2.3 Case(a): Temperature and Concentration

Assume that the exponentially stretching surface is maintained at the stretching velocity $u_w(x)$, exponential temperature distribution $T_w(x)$ and exponential concentration distribution $C_w(x)$, which are defined by

$$u_w(x) = u_0 e^{x/L}, \tag{2.5}$$

$$T_w(x) = T_\infty + (T_0 - T_\infty)e^{x/2L}, \tag{2.6}$$

$$C_w(x) = C_\infty + (C_0 - C_\infty)e^{x/2L}, \tag{2.7}$$

where T_0 is a parameter of the temperature distribution where as C_0 is a parameter of the concentration distribution in the stretching surface.

Hence, the boundary conditions are

$$u = u_w(x), \;\; v = 0, \;\; T = T_w(x), \;\; C = C_w(x) \quad at \quad y = 0, \tag{2.8a}$$

$$u = 0, \;\; T = T_\infty, \;\; C = C_\infty \quad as \quad y \to \infty, \tag{2.8b}$$

where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively.

In view of the continuity equation (2.1), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{2.9}$$

Substituting (2.9) in (2.2)-(2.4) and then using the following local-similarity transformations

$$\left.\begin{aligned} &\eta = \left(\frac{Re}{2}\right)^{1/2} \frac{y}{L} e^{x/2L}, \quad \psi = \sqrt{2}\nu Re^{1/2} e^{x/2L} f(\eta), \\ &T(x,y) = T_\infty + (T_0 - T_\infty) e^{x/2L} \theta(\eta), \quad C(x,y) = C_\infty + (C_0 - C_\infty) e^{x/2L} \phi(\eta), \end{aligned}\right\} \tag{2.10}$$

where $Re_x = \dfrac{u_w(x)x}{\nu}$ is the local Reynolds number, we get the following local-similarity equations

$$f''' + ff'' - 2f'^2 + 2Ri\ e^{-3X/2} (\theta + \mathcal{B}\phi) - 2M\ e^{-X} f' = 0 \tag{2.11}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta + D_f\phi'' = 0 \tag{2.12}$$

$$\frac{1}{Sc}\phi'' + f\phi' - f'\phi + S_r\theta'' = 0, \tag{2.13}$$

where the primes indicate partial differentiation with respect to η alone, ν is the kinematic viscosity, L is the characteristic length of the plate, $X = \dfrac{x}{L}$ is the X-location, $Gr = \dfrac{g^*\beta_T(T_0 - T_\infty)L^3}{\nu^2}$ is the thermal Grashof number, $Gc = \dfrac{g^*\beta_C(C_0 - C_\infty)L^3}{\nu^2}$ is the solutal Grashof number, $Re = \dfrac{u_0 L}{\nu}$ is the Reynolds number, $Ri = \dfrac{Gr}{Re^2}$ is the mixed convection parameter, $N = \dfrac{\beta_C(C_0 - C_\infty)}{\beta_T(T_0 - T_\infty)}$ is the buoyancy ratio, $Ha^2 = \dfrac{\sigma B_0^2 L^2}{\rho\nu}$ is the Hartman number, $M = \dfrac{Ha^2}{Re}$ is the magnetic parameter which is the ratio of Hartman number to Reynolds number, $Pr = \dfrac{\nu}{\alpha}$ is the Prandtl number, $Sc = \dfrac{\nu}{D}$ is the Schmidt number, $D_f = \dfrac{DK_T(C_0 - C_\infty)}{C_s C_p \nu(T_0 - T_\infty)}$ is the Dufour number and $S_r = \dfrac{DK_T(T_0 - T_\infty)}{T_m \nu(C_0 - C_\infty)}$ is the Soret number. Same symbols are used throughout the thesis unless otherwise specified.

Boundary conditions (2.8) in terms of f, θ and ϕ become

$$\eta = 0: \ f(0) = 0, \ f'(0) = 1, \ \theta(0) = 1, \ \phi(0) = 1 \tag{2.14a}$$

$$\eta \to \infty: \ f'(\infty) = 0, \ \theta(\infty) = 0, \ \phi(\infty) = 0. \tag{2.14b}$$

A close look at Eqs.(2.11) reveals that, in mixed convection due to viscous fluid, the velocity profile are not similar because the x-coordinate cannot be eliminated from this equation. Although local non-similarity solutions have been found for some boundary layer flows dealing with a viscous fluid, the technique is hard to extend to in this case. Thus, for ease of analysis, it was decided to proceed with finding local similarity solutions for the governing Eqs. (2.11). That is, taking $X = \frac{x}{L}$ and then varying the X-location, one can still study the effects of various parameters on different profiles at any given X-location.

The results of practical interest in many applications are the skin friction, heat and mass transfer coefficients. The wall shear stress, heat and mass transfers from the surface respectively are given by

$$\tau_w = \mu \left[\frac{\partial u}{\partial y}\right]_{y=0}, \quad q_w(x) = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad \text{and} \quad q_m(x) = -D\left[\frac{\partial C}{\partial y}\right]_{y=0},$$

where k is the thermal conductivity.

The non-dimensional skin friction $C_f = \frac{2\tau_w}{\rho U_*^2}$, the local Nusselt number $Nu_x = \frac{q_w(x)x}{k(T_w(x) - T_\infty)}$ and local Sherwood number $Sh_x = \frac{q_m(x)x}{D(C_w(x) - C_\infty)}$, where U_* is the characteristic velocity, are given by

$$C_f\sqrt{Re_x} = \sqrt{2X} \quad f''(0), \tag{2.15a}$$

$$\frac{Nu_x}{\sqrt{Re_x}} = -\sqrt{\frac{X}{2}} \quad \theta'(0), \qquad \frac{Sh_x}{\sqrt{Re_x}} = -\sqrt{\frac{X}{2}} \quad \phi'(0). \tag{2.15b}$$

2.4 Results and Discussions

The system of nonlinear ordinary differential Eqs. (2.11) - (2.13) together with the boundary conditions (2.14) are locally similar and solved numerically using the shooting and matching technique. The calculations are repeated until some convergence criterion is satisfied and the calculations were stopped when $\delta f_0'' \leq 10^{-8}$, $\delta\theta_0' \leq 10^{-8}$ and $\delta\phi_0' \leq 10^{-8}$. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity, temperature and concentration approach zero. In order to see the effects of step size ($\Delta\eta$), the code for our model was run with three different step sizes as $\Delta\eta = 0.001$, $\Delta\eta = 0.01$ and $\Delta\eta = 0.05$ and in each case very good agreement between them on different profiles was found. After some trials, a maximal value of η at ∞ of 15 and a grid size of η as 0.01 was improved.

In addition, in order to verify the accuracy of the present numerical method, the results are

Table 2.1: *Comparison between wall-temperature gradient $\theta'(0)$ calculated by the present method and that of Magyari and Keller [13] for $M = Ri = S_r = D_f = N = 0$.*

Pr	Magyari and Keller[13]	Present
0.5	-0.59434	-0.59434
1.0	-0.95478	-0.95478
3.0	-1.86908	-1.86908
5.0	-2.50014	-2.50014
8.0	-3.24213	-3.24213
10.0	-3.66038	-3.66038

compared with those reported earlier by Magyari and Keller [13]. The results of these comparisons are shown in Table (2.1). It can be seen from this table that excellent agreement between the results exist. This lends confidence in the numerical results reported below. In order to study the effects of magnetic parameter M, mixed convection parameter Ri, Soret number S_r, Dufour number D_f and X-location explicitly, computations were carried out for the cases of $Pr = 1.0, Sc = 0.22$ and $N = 0.5$.

The variation of the non-dimensional velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ for $Ri = 5.0, S_r = 2.0, D_f = 0.03$ and $X = 0.5$ with magnetic parameter is shown in Fig. 2.2(a)-2.2(c). It can be observed from Fig. 2.2(a) that the velocity of the fluid is decreases with an increase in the value of the magnetic parameter. This is due to the fact that, the introduction of a transverse magnetic field, normal to the flow direction, has a tendency to create the drag known as the Lorentz force which tends to resist the flow. Hence the horizontal velocity profiles decrease as the magnetic parameter M increases. It can be found from Fig. 2.2(b) that increase in the value of the magnetic parameter increases the temperature of the fluid in the medium. It can be seen from Fig. 2.2(c) that the concentration of the fluid is increased by increasing the value of the magnetic parameter. As explained above, the transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and thus increases its temperature and concentration.

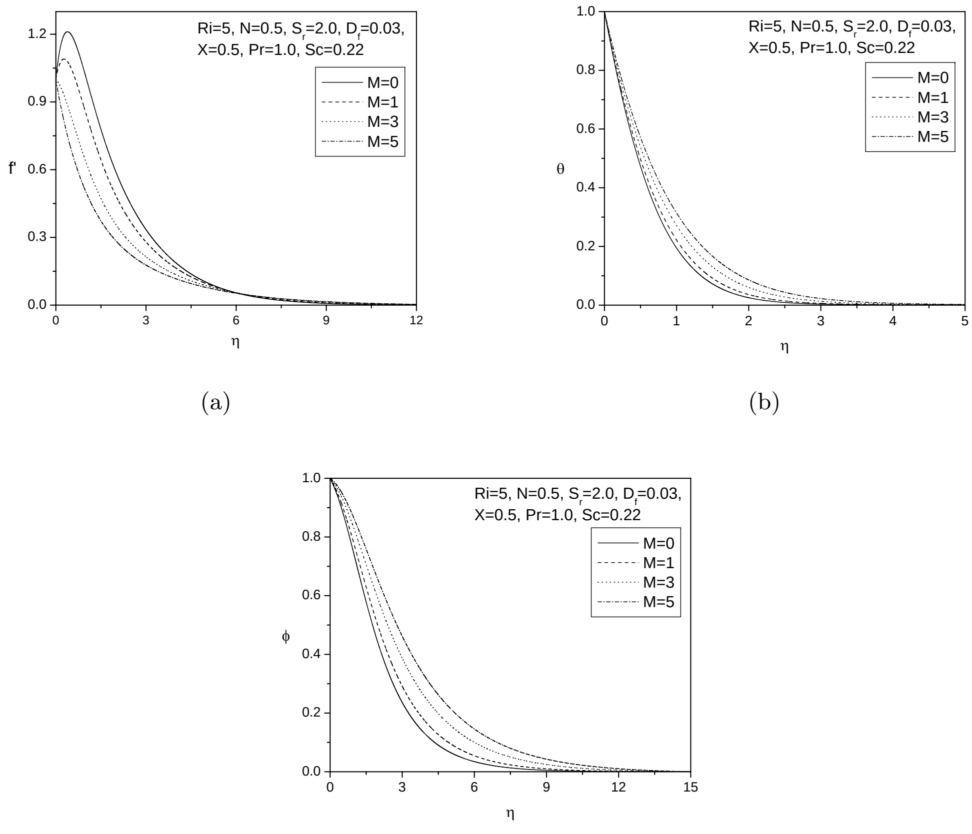
Fig. 2.3(a) shows the dimensionless velocity profile for various values of the mixed convection parameter Ri with fixed values of M, S_r, D_f and X. It reveals that as the value of Ri increases, the dimensionless velocity rises. Compared with the limiting case of $Ri = 0.0$ (i.e., pure forced convection), an increase in the value of Ri gives rise to a higher velocity. Since a greater value of Ri

indicates a greater buoyancy effects in mixed convection flow leads to an acceleration of the fluid flow. Fig. 2.3(b) illustrates the dimensionless temperature for selected values of Ri. The results indicate that the dimensionless temperature reduces with the increase of Ri. The temperature in case of mixed convection is less compared to that of pure forced convection). As Ri (i.e., buoyancy effects) increase, the convection cooling effect increases and hence the temperature reduces. The effect of mixed convection parameter Ri on the dimensionless concentration is depicted in Fig. 2.3(c). It is clear that the concentration of the fluid decreases with the increase of mixed convection parameter Ri.

Fig. 2.4(a) displays the non-dimensional velocity for different values of Soret number S_r and Dufour number D_f with $Ri = 5.0, M = 0.5$ and $X = 0.5$. It is observed that the velocity of the fluid increases with the increase of the Dufour number (or a decrease of Soret number). The dimensionless temperature for different values of Soret number S_r and Dufour number D_f, is shown in Fig. 2.4(b). It is clear that the thickening of the thermal boundary layer with the increase of the Dufour number (or a decrease Soret number). Fig. 2.4(c) demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f. The mass flux created by the temperature gradient gives rise to Soret or thermal-diffusion or thermophoresis effect. The thermophoretic force developed due to temperature gradients drives solute particles into the boundary layer region thereby increasing the concentration boundary layer as can be seen from Fig. 2.4(c) with increasing values of Soret number(or a decrease of Dufour number). Hence, the cross-diffusion effect plays an important role in enhancing heat and mass transfer in double-diffusive convection processes.

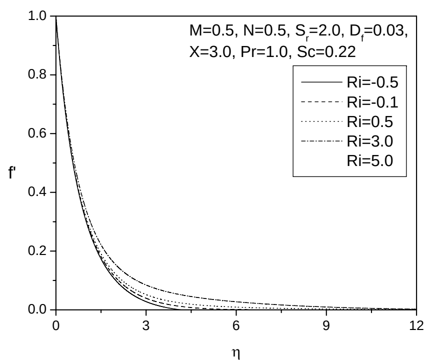
In Figs. 2.5(a)-2.5(c), the effects of the X-location on the dimensionless velocity, temperature and concentration are presented for fixed values of M, S_r, D_f and X. From the Fig. 2.5(a), it is noticed that the velocity decreases with an increase in the value of X in the momentum boundary layer. It is clear from Fig. 2.5(b) that the thermal boundary layer thickness increases with the increase of X. It can be seen from Fig. 2.5(c) that the solutal boundary layer thickness of the fluid increases with the increase of X.

The variations of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ which are proportional to the local skin-friction coefficient, rate of heat and mass transfers are shown in Table(2.2) for different values of the mixed convection parameter Ri with fixed values of Sr, Df, M and X-location in both cases of opposing and aiding flows. Table(2.2) illustrates the effects of the mixed convection parameter Ri on the skin friction coefficient $f''(0)$. It is seen that the local skin friction factor increases as Ri increases. The reason is that an increase in the buoyancy effect in mixed convection flow leads to an acceleration of the fluid flow, which increases the local skin friction factor. It is seen from this table that, the heat and mass transfer rates increase in both cases of opposing and aiding flows with the increasing value of Ri. Hence the mixed convection parameter has an important role in controlling the temperature and concentration. It is depicted from Table(2.2) that, for fixed values of Sr, Df, Ri and M, the skin friction, heat and mass transfer coefficients are reduced with the increasing value of X-location. The effect of increasing the value of M is to decrease the skin friction, heat and mass transfer coefficients with fixed values of Sr, Df, Ri and X-location. Finally, the effects of Dufour and Soret number on the local skin-friction coefficient and the rate of heat and mass transfer are shown in this table. The behavior of these parameters is self-evident from the Table(2.2) and hence are not discussed for brevity.

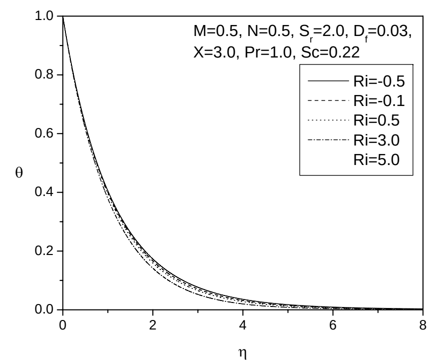


(a) (b) (c)

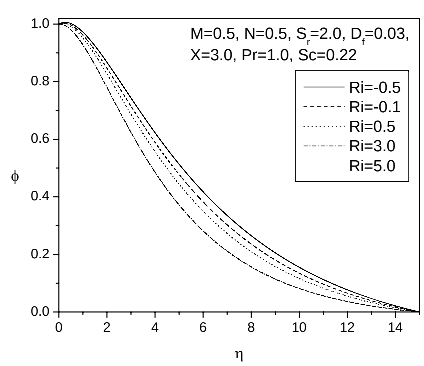
Figure 2.2: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of M*



(a)

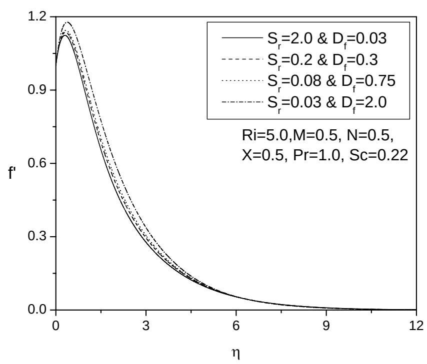


(b)

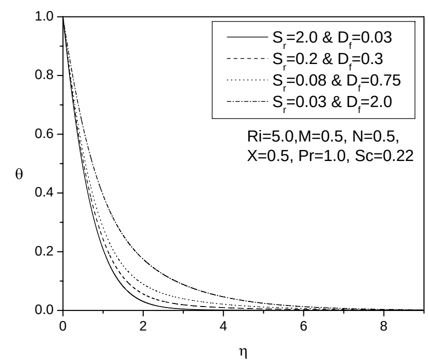


(c)

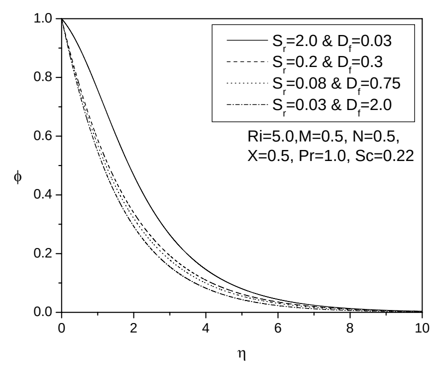
Figure 2.3: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of Ri*



(a)

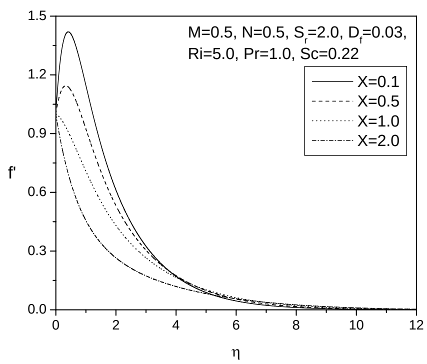


(b)

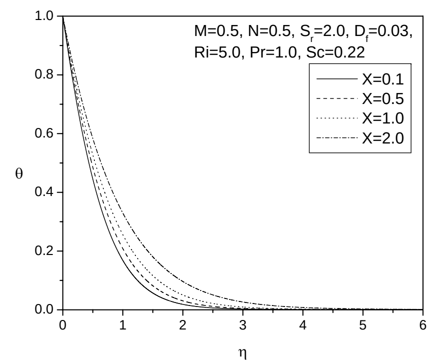


(c)

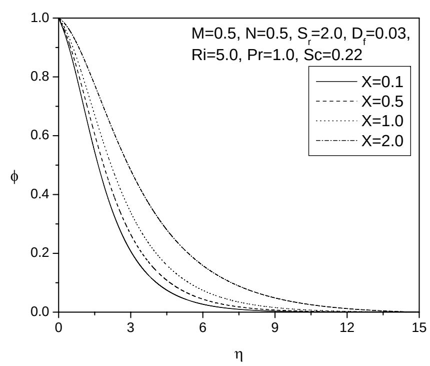
Figure 2.4: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of* S_r *and* D_f



(a)



(b)



(c)

Figure 2.5: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of X*

Table 2.2: *Effects of skin friction, heat and mass transfer coefficients for varying values of* Ri, Sr, Df, X *and* M *with* $Pr = 1.0,\ Sc = 0.22$ *and* $N = 0.5$.

Ri	S_r	D_f	X	M	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
-0.5	2.0	0.03	3.0	0.5	-1.31304	0.94392	-0.04920
-0.1	2.0	0.03	3.0	0.5	-1.30370	0.95015	-0.03888
0.5	2.0	0.03	3.0	0.5	-1.29087	0.95719	-0.02887
3.0	2.0	0.03	3.0	0.5	-1.24272	0.97758	-0.00590
5.0	2.0	0.03	3.0	0.5	-1.20704	0.98986	0.00528
5.0	2.0	0.03	0.5	0.5	1.01268	1.29649	0.14317
5.0	1.6	0.0375	0.5	0.5	0.99232	1.29096	0.22346
5.0	1.2	0.05	0.5	0.5	0.97240	1.28441	0.30358
5.0	1.0	0.06	0.5	0.5	0.96280	1.28039	0.34363
5.0	0.8	0.075	0.5	0.5	0.95371	1.27542	0.38372
5.0	0.5	0.12	0.5	0.5	0.94246	1.26373	0.44423
5.0	0.2	0.3	0.5	0.5	0.94568	1.22681	0.50748
5.0	0.1	0.6	0.5	0.5	0.97240	1.16952	0.53336
5.0	2.0	0.03	0.1	0.5	2.51423	1.41108	0.17661
5.0	2.0	0.03	0.5	0.5	1.01268	1.29649	0.14317
5.0	2.0	0.03	1.0	0.5	-0.07811	1.18615	0.10624
5.0	2.0	0.03	2.0	0.5	-0.96903	1.05074	0.04682
5.0	2.0	0.03	0.5	0.0	1.29431	1.32878	0.15978
5.0	2.0	0.03	0.5	1.0	0.75350	1.26619	0.12827
5.0	2.0	0.03	0.5	3.0	-0.11357	1.16200	0.08308
5.0	2.0	0.03	0.5	5.0	-0.79641	1.07892	0.05430

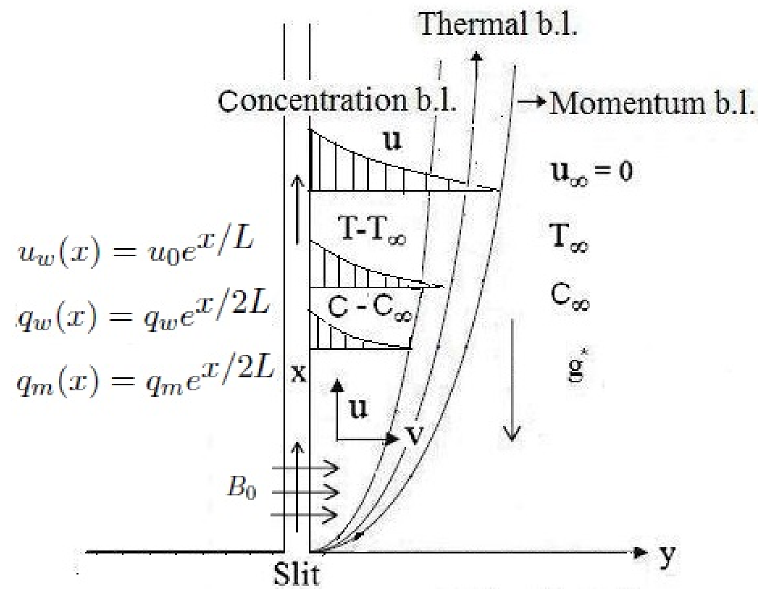


Figure 2.6: *Physical model and coordinate system.*

2.5 Case(b): Heat and Mass Fluxes

Assume that the exponentially stretching surface is maintained at the stretching velocity $u_w(x)$, exponential heat flux $q_w(x)$ and exponential mass flux $q_m(x)$, which are defined by

$$u_w(x) = u_0 e^{x/L}, \quad q_w(x) = q_w e^{x/2L} \quad \text{and} \quad q_m(x) = q_m e^{x/2L}, \tag{2.16}$$

where u_0 is a parameter of the velocity, q_w is a parameter of the temperature distribution where as q_m is a parameter of the concentration distribution in the stretching surface.

The physical model and coordinate system are shown in Fig.(2.6). The steady mixed convection boundary layer flow is governed by Eqns. (2.1) to (2.4) and the boundary conditions are given by

$$u = u_w(x), \;\; v = 0, \;\; -k\left(\frac{\partial T}{\partial y}\right)_w = q_w(x), \;\; -D\left(\frac{\partial C}{\partial y}\right)_w = q_m(x) \quad at \quad y = 0, \tag{2.17a}$$

$$u = 0, \;\; T = T_\infty, \;\; C = C_\infty \quad as \quad y \to \infty, \tag{2.17b}$$

where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively,

Substituting the stream function ψ given by Eqn. (2.9) in (2.2)-(2.4) and then using the following local-similarity transformations

$$\left.\begin{aligned}&\eta=\left(\frac{Re}{2}\right)^{1/2}\frac{y}{L}e^{x/2L}, \quad \psi=\sqrt{2}\nu Re^{1/2}e^{x/2L}f(\eta),\\ &T(x,y)=T_\infty+\frac{\sqrt{2}q_wL}{k}Re^{-1/2}e^{x/2L}\theta(\eta), \quad C(x,y)=C_\infty+\frac{\sqrt{2}q_mL}{D}Re^{-1/2}e^{x/2L}\phi(\eta),\end{aligned}\right\} \tag{2.18}$$

we get the following nonlinear system of differential equations:

$$f'''+ff''-2f'^2+2Ri\ e^{-3X/2}(\theta+N\phi)-2M\ e^{-X}f'=0 \tag{2.19}$$

$$\frac{1}{Pr}\theta''+f\theta'-f'\theta+D_f\phi''=0 \tag{2.20}$$

$$\frac{1}{Sc}\phi''+f\phi'-f'\phi+S_r\theta''=0, \tag{2.21}$$

where the primes indicate partial differentiation with respect to η alone, $Gr=\dfrac{g^*\beta_Tq_wL^4}{k\nu^2}$ is the thermal Grashof number, $Ri=\dfrac{Gr}{Re^{5/2}}$ is the mixed convection parameter, $N=\dfrac{\beta_Cq_mk}{\beta_Tq_wD}$ is the buoyancy ratio, $D_f=\dfrac{DK_T}{C_sC_p\nu}\dfrac{q_mk}{q_mD}$ is the Dufour number and $S_r=\dfrac{DK_T}{T_m\nu}\dfrac{q_wD}{q_wk}$ is the Soret number.

Boundary conditions (2.17) in terms of f, θ and ϕ become

$$\eta=0:\ f(0)=0,\ f'(0)=1,\ \theta'(0)=-1,\ \phi'(0)=-1, \tag{2.22a}$$

$$\eta\to\infty:\ f'(\infty)=0,\ \theta(\infty)=0,\ \phi(\infty)=0. \tag{2.22b}$$

The non-dimensional skin friction C_f, the local Nusselt number Nu_x and local Sherwood number Sh_x are given by

$$C_f\sqrt{Re_x}=\sqrt{2X}\quad f''(0), \tag{2.23a}$$

$$\frac{Nu_x}{\sqrt{Re_x}}=\frac{X}{\sqrt{2}}e^{X/2}\frac{1}{\theta(0)}, \quad \frac{Sh_x}{\sqrt{Re_x}}=\frac{X}{\sqrt{2}}e^{X/2}\frac{1}{\phi(0)}. \tag{2.23b}$$

2.6 Results and Discussions

The reduced governing equations (2.19) to (2.21) along with the boundary conditions (2.22) are solved numerically using the shooting method as in the previous case. The value of η_∞ is taken as

15 and a grid size of η as 0.01. Also, the solutions for the dimensionless velocity, temperature and concentration function are shown graphically in Figs. 2.7(a)-2.10(c). The values of Pr, Sc and N are chosen to be same as in the case(a).

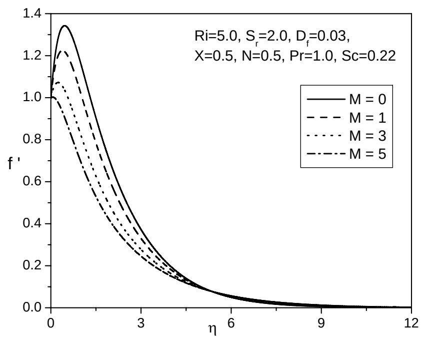
The effects of the M on the dimensionless velocity, temperature and concentration are depicted for fixed values of Ri, Sr, Df and X-location in the Figs. 2.7(a)-2.7(c). It can be seen from Fig. 2.7(a) that the velocity reduces As M enhances. Fig. 2.7(b) indicates that a rise in M enhances the temperature and concentration in the viscous fluid as in the case(a).

Fig. 2.8(a) shows the dimensionless velocity profile for various values of the mixed convection parameter Ri with fixed values of Sr, Df, M and X-location. It reveals that as the value of Ri increases, the dimensionless velocity rises. Compared with the limiting case of $Ri = 0.0$ (i.e., pure forced convection), the velocity is more for aiding flow and the velocity is less for opposing flow. Figs. 2.8(b) and 2.8(c) illustrate the dimensionless temperature and concentration for different values of Ri. The results indicate that the dimensionless temperature and concentration reduce with the increase of Ri.

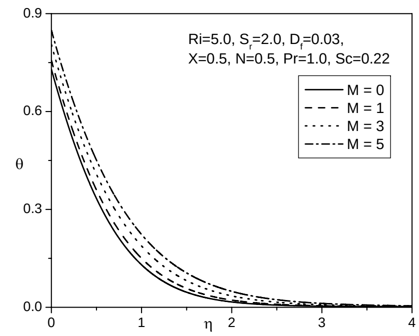
Fig.2.9(a) displays the non-dimensional velocity for different values of Soret number S_r and Dufour number D_f with $Ri = 1.0, X = 0.5, M = 0.5$. It is observed that the velocity of the fluid decreases with smaller values D_f and increases with larger values of Dufour number (or a decrease of Soret number). The dimensionless temperature for different values of Soret number S_r and Dufour number D_f for $Ri = 1.0, X = 0.5, M = 0.5$, is shown in Fig.2.9(b). It is clear that the temperature of the fluid increases with the increase of the Dufour number (or decrease Soret number). Fig.2.9(c) demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f for $Ri = 1.0, X = 0.5, M = 0.5$. It is seen that the concentration of the fluid decreases with increase of the Dufour number (or a decrease of Soret number).

In Figs. 2.10(a)-2.10(b), the effects of the X-location on the dimensionless velocity, temperature and concentration are presented for fixed values of Ri, Sr, Df and M. From the Fig. 2.10(a), it is noticed that the velocity decreases with an increase in the value of X in the momentum boundary layer. It is clear from Fig. 2.10(b) that the thermal boundary layer thickness increases with the increase of X but with significant effect near the stretching sheet. It can be seen from Fig. 2.10(c) that the solutal boundary layer thickness of the fluid increases with the increase of X and also, found that significant effect within the boundary layer.

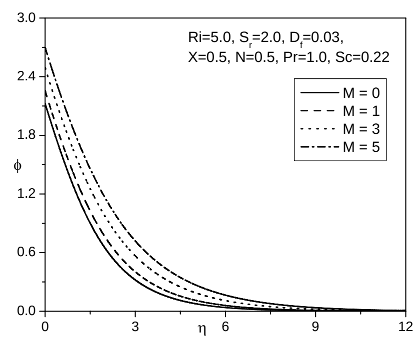
The variations of the skin friction,local heat transfer and the local mass transfer coefficients for various values of the mixed convection parameter Ri with $M = 0.5, S_r = 2.0, D_f = 0.03, X = 0.5$, are shown in Table(2.3). It is seen that the local skin friction factor, the heat and mass transfer rates increase with the increasing value of Ri. Table(2.3) depict the variations of the skin friction coefficient $f''(0)$, local heat transfer coefficient $\frac{1}{\theta(0)}$ and the local mass transfer coefficient $\frac{1}{\phi(0)}$ for various values of the magnetic number M with $Ri = 5.0, S_r = 2.0, D_f = 0.03, X = 0.5$. It is observed that the skin friction, local heat transfer and the local mass transfer decrease with increasing values of the magnetic number M. Again, the skin friction, local heat and mass transfer coefficients are predicted to increase as the X-location increases. Table(2.3) illustrate the variations of the skin friction,local heat transfer and the local mass transfer coefficients for various values of the Soret number S_r and Dufour number D_f with $M = 0.5, Ri = 5.0, X = 0.5$. It is to be noted from Table(2.3) that simultaneously increasing D_f and decreasing S_r lead to initial decreases in the skin-friction coefficient $f''(0)$ up to $S_r = 0.12$, $D_f = 0.5$ and then start increasing. The heat transfer coefficient $-\theta'(0)$ shows a monotonic decrease, while the mass transfer coefficient $-\phi'(0)$ exhibits the opposite change when subjected to simultaneous increase in D_f and decrease in S_r. This is due to the coupling between the momentum, energy, and species balance equations, the Dufour parameter has an effect on the concentration boundary layer as well.



(a)

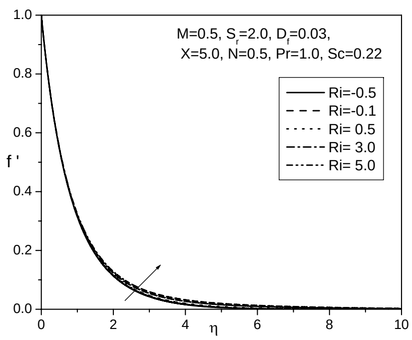


(b)

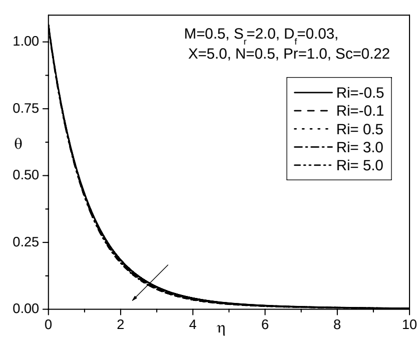


(c)

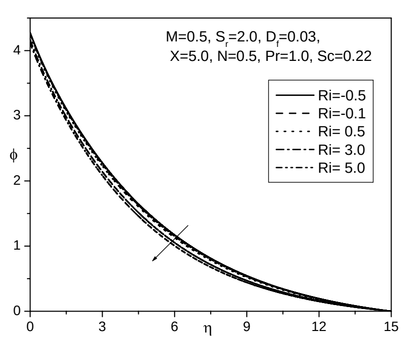
Figure 2.7: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of M*



(a)

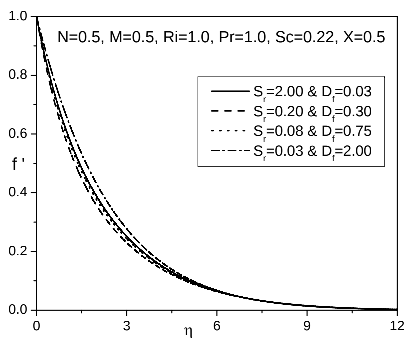


(b)

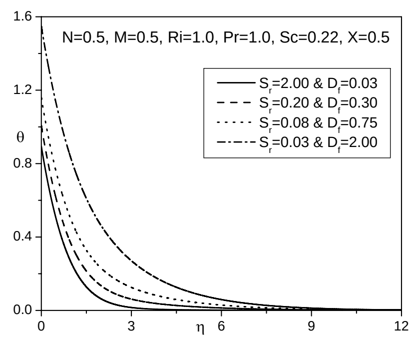


(c)

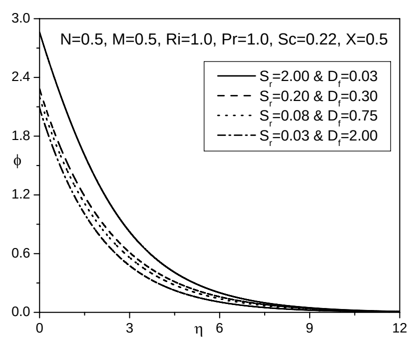
Figure 2.8: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of Ri*



(a)

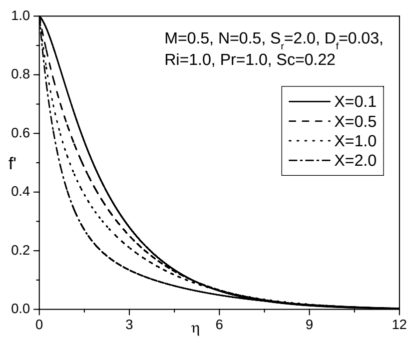


(b)

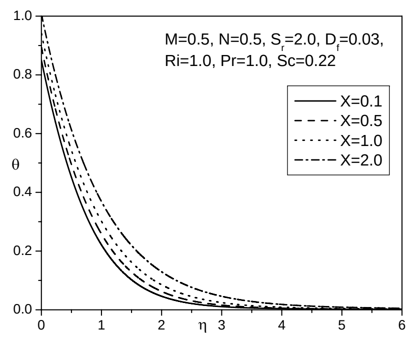


(c)

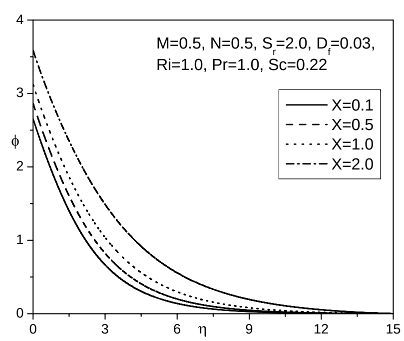
Figure 2.9: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of* S_r *and* D_f



(a)



(b)



(c)

Figure 2.10: *(a)Velocity, (b)Temperature and (c)Concentration profiles for various values of X*

Table 2.3: *Effects of skin friction, heat and mass transfer coefficients for varying values of* Ri, Sr, Df, X *and* M *with* $Pr = 1.0$, $Sc = 0.22$ *and* $N = 0.5$.

Ri	S_r	D_f	X	M	$f''(0)$	$\frac{1}{\theta(0)}$	$\frac{1}{\phi(0)}$
-0.5	2.0	0.03	5.0	0.5	-1.28588	0.93903	0.23390
-0.1	2.0	0.03	5.0	0.5	-1.28477	0.93984	0.23477
0.5	2.0	0.03	5.0	0.5	-1.28313	0.94100	0.23599
3.0	2.0	0.03	5.0	0.5	-1.27671	0.94518	0.24023
5.0	2.0	0.03	5.0	0.5	-1.27188	0.94803	0.24300
1.0	2.0	0.03	0.5	0.5	-0.53753	1.11854	0.34934
1.0	1.6	0.0375	0.5	0.5	-0.56324	1.11032	0.36464
1.0	1.2	0.05	0.5	0.5	-0.58869	1.09985	0.38165
1.0	1.0	0.06	0.5	0.5	-0.60105	1.09302	0.39092
1.0	0.8	0.075	0.5	0.5	-0.61283	1.08413	0.40082
1.0	0.5	0.12	0.5	0.5	-0.62750	1.06200	0.41726
1.0	0.2	0.3	0.5	0.5	-0.62330	0.99261	0.43820
1.0	0.1	0.6	0.5	0.5	-0.58869	0.90120	0.45057
1.0	2.0	0.03	0.1	0.5	-0.09620	1.17603	0.37676
1.0	2.0	0.03	0.5	0.5	-0.53753	1.11854	0.34934
1.0	2.0	0.03	1.0	0.5	-0.87651	1.06237	0.32052
1.0	2.0	0.03	2.0	0.5	-1.17274	0.99194	0.27878
5.0	2.0	0.03	0.5	0.0	1.83496	1.37049	0.47057
5.0	2.0	0.03	0.5	1.0	1.39904	1.32190	0.44343
5.0	2.0	0.03	0.5	3.0	0.68946	1.24175	0.40139
5.0	2.0	0.03	0.5	5.0	0.11695	1.17784	0.37051

2.7 Conclusions

In this chapter, we have studied the problem of steady, two-dimensional and laminar flow of an electrically conducting and incompressible viscous fluid near an impermeable exponentially stretching surface with (a) temperature and concentration conditions and (b) heat and mass flux conditions, in the presence of Soret and Dufour effects.

From this study we can draw the following conclusions for both the cases (a) and (b): The higher values of the magnetic parameter M (i.e., for the case where the effect of MHD becomes significant) result in lower velocity distribution but higher wall temperature, wall concentration distributions in the boundary layer. The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfers in the MHD Newtonian fluid are lower compared to those of the Newtonian fluid without magnetic field. The higher values of the mixed convection parameter Ri indicates the lower velocity distribution and but higher wall temperature, wall concentration distributions. Also, the skin friction coefficient as well as rates of heat and mass transfer increase as mixed convection parameter Ri increases. An increase in D_f (or decrease in S_r), increases velocity, wall temperature and mass transfer rate but decreases wall concentration as well as the rate of heat transfer. Further, the skin friction coefficient decreases and then increases with an increase in D_f (or decrease in S_r). The velocity and wall temperature distributions increase whereas the wall concentration distribution, skin friction coefficient as well as the rate of heat and mass transfers decrease with an increase in the value of X in the boundary layer.

Chapter 3

Summary and Conclusions

The study of convective transport in a Newtonian fluid not only possesses the theoretical appeal but also finds application in diverse engineering and industrial systems. The problem of mixed convection heat and mass transfer near an exponentially stretching surface in a viscous fluid in the presence of combined magnetic field, and Soret and Dufour effects is studied in Chapter-2. Using a set of suitable local-similarity variables, the governing equations are transformed into a set of ordinary differential equations depending on several dimensionless parameters. Between these parameters, there is also the dimensionless distance along the plate, and this approach is a characteristic feature of the present investigation. The main results indicate the following findings:

- The higher values of the magnetic parameter M result in lower velocity distribution but higher skin friction coefficient, heat and mass transfer rates, and temperature and concentration distributions.

- The present analysis has also shown that an increase in the Dufour number (or decrease in Soret number), increase the velocity, wall temperature and mass transfer rate but decreases wall concentration as well as the rate of heat transfer. The skin friction coefficient decreases and then increases with an increase in the Dufour number (or decrease in Soret number).

- Higher values of the mixed convection parameter indicate lower velocity distribution but higher temperature and concentration distributions. Also, the skin friction coefficient as well as rates of heat and mass transfer increase as mixed convection increases.

Bibliography

[1] C. R. A. Abreu, M. F. Alfradique, and A. T. Silva. Boundary layer flows with Dufour and Soret effects: I: Forced and natural convection. *Chemical Engineering Science*, 61:4282–4289, 2006.

[2] S. Alam, M.M. Rahman, A. Maleque, and M. Ferdows. Dufour and Soret effects on steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. *Thammasat Int. J. Tech.*, 11:1–12, 2006.

[3] A. Bejan. *Convection Heat Transfer*. John Wiley., New York, 2004.

[4] B. Bidin and R. Nazar. Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. *European Journal of Scientific Research*, 33:710–717, 2009.

[5] T. S. Chen and B. F. Armaly. *Mixed Convection in External FLow. In Kakac, S., Shah, R.K. and Aung, W.eds. Handbook of Single-Phase Convective Heat Transfer*. John Wiley., New York, 1987.

[6] Z. Dursunkaya and W. M. Worek. Diffusion-thermo and thermal diffusion effects in transient and steady natural convection from a vertical surface. *Int. J. Heat Mass Transfer*, 35:2060–2065, 1992.

[7] E. R. G. Eckeret and R. M. Drake. *Analysis of heat and mass transfer*. McGraw Hill, New York, 1972.

[8] E. M. A. Elbahbeshy. Heat transfer over an exponentially stretching continuous surface with suction. *Arch. Mech.*, 53:643–651, 2001.

[9] A. Ishak. MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. *Sains Malaysiana*, 40:391–395, 2011.

[10] R. N. Jat and C. Gopi. MHD flow and heat transfer over an exponentially stretching sheet with viscous dissipation and radiation effects. *Applied Mathematical Sciences*, 7:167 – 180, 2013.

[11] N. G. Kafoussias and N. G. Williams. Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. *Int. J. Engg. Sci.*, 33:1369–1384, 1995.

[12] P. A. Lakshmi Narayana and P. V. S. N. Murthy. Soret and Dufour effects in a doubly stratified Darcy porous medium. *Journal of Porous Media*, 10:613624, 2007.

[13] E. Magyari and B. Keller. Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *J. Phys. D: Appl. Phy.*, 32:577–585, 1999.

[14] D. A. Nield and A. Bejan. *Convection in Porous Media, 3rd Edition*. Springer-Verlag, New York, 2006.

[15] D. Pal. Mixed convection heat transfer in the boundary layers on an exponentially stretching surface with magnetic field. *Applied Mathematics and Computation*, 217:2356–2369, 2010.

[16] M. K. Partha, P. V. S. N. Murthy, and G. P. Rajasekhar. Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. *Heat Mass Transfer*, 41:360–366, 2005.

[17] A. Postelnicu. Influence of a magnetic field on heat and mass transfer by natural convection from vertical sufaces in porous media considering Soret and Dufour effects. *Int. J. Heat Mass Transfer*, 47:1467–1475, 2004.

[18] Ch. RamReddy and D. Srinivasacharya. Soret and Dufour effects on mixed convection from an exponentially stretching surface in non-Darcy porous medium. *2012 International Conference on Fluid Dynamics and Thermodynamics Technologies[IACSIT Press, Singapore]*, 33:43–48, 2012.

[19] A. Raptis and A. K. Singh. MHD free convection flow past an accelerated vertical plate. *Int. Commun. Heat Mass Transfer*, 10:313–321, 1983.

[20] S. Rawat and R. Bhargava. Finite element study of natural convection heat and mass transfer in a micropolar fluid saturated porous regime with Soret/Dufour effects. *Int. J. of Appl. Math and Mech.*, 5:58–71, 2009.

[21] M.Z. Saghir, C.G. Jiang, M. Chacia, Y. Yan, M. Khawaja, and S. Pan. *Thermodiffusion in Porous Media(In Ingham, D., Pop, I. (eds)), Transport Phenomena in Porous Media III, 9th chapter.* Pergamon, Oxford, 2005.

[22] M. Sajid and T. Hayat. Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. *Int. Commun. Heat Mass Transfer*, 35:347–356, 2008.

[23] B. C. Sakiadis. Boundary-layer behavior on continuous solid surfaces. *AIChE J.*, 7:26–28, 1961.

[24] C. Soret. Influence de la temperature sur la distribution des sels dans leurs solutions. *C.R. Acad. Sci., Paris*, 91:289–291, 1880.

[25] D. Srinivasacharya, J. Pranitha, and Ch. RamReddy. Magnetic effect on non-Darcy free convection in a power-law fluid with double stratification. *J. Braz. Soc. Mech. Sci. and Eng.*, 33:8–14, 2011.

[26] D. Srinivasacharya, J. Pranitha, and Ch. RamReddy. Magnetic and double dispersion effects on free convection in a non-Darcy porous medium saturated with power-law fluid. *Int. J. Comput. Methods in Engg. Science and Mechanics*, 13:210–218, 2012.

[27] D. Srinivasacharya and Ch. RamReddy. Effects of Soret and Dufour on mixed convection heat and mass transfer in a micropolar fluid with heat and mass fluxes. *Int. J. of Appl. Math and Mech.*, 6:8–21, 2010.

[28] D. Srinivasacharya and Ch. RamReddy. Mixed convection from an exponentially stretching surface with Soret and Dufour effects. *IEEE proceedings of 2011 World Congress on Engineering and Technology, China*, 2:707–711, 2011.

[29] D. Srinivasacharya and Ch. RamReddy. Mixed convection heat and mass transfer in a micropolar fluid with Soret and Dufour effects. *Adv. Appl. Math. Mech.*, 3:389–400, 2011.

[30] D. Srinivasacharya and Ch. RamReddy. Soret and Dufour effects on mixed convection from an exponentially stretching surface. *Int. Journal of Nonlinear Science*, 12:60–68, 2011.

[31] D. Srinivasacharya and Ch. RamReddy. Cross-diffusion effects on mixed convection from an exponentially stretching surface in non-Darcy porous medium. *Heat Transfer - Asian Journal (Wiley)*, DOI 10.1002/htj.21032:1–14, 2013.